sent memoir. I have, however, excluded from the Tables, and I do not in the memoir consider (otherwise than incidentally) the covariant of the sixth order ΘU, or the contravariant (reciprocant) FU.

I have given in the memoir a comparison of my notation with that of M. Aronhold. A short part of the memoir relates to the binary cubic and the binary quartic, viz. each of these quantics has a covariant of its own order, forming with it an intermediate $\alpha U+\beta W$, the covariants whereof contain quantics in (α, β), the coefficients of which are invariants of the original quantic. The formulæ which relate to these cases are in fact given in my Fifth Memoir, but they are reproduced here in order to show the relations between the quantics in (α, β) contained in the formulæ. As regards the binary quartic, these results are required for the discussion of the like question in regard to the ternary cubic, viz. that of finding the relations between the different quantics in (α, β) contained in the formulæ relating to the ternary cubic. Some of these relations have been obtained by M. Hermite in the memoir "Sur les formes cubiques à trois indéterminées," Liouville, t. iii. pp. 37–40 (1858), and in that " Sur la Résolution des équation du quatrième degré," Comptes Rendus, t. xlvi. p. 715 (1858), and by M. Aronhold in his memoir already referred to; and in particular I reproduce and demonstrate some of the results in the last mentioned memoir of M. Hermite. But the relations in question are in the present memoir exhibited in a more complete and systematic form.

III. "On the Secular Change in the Magnetic Dip in London, between the years 1821 and 1860." By Major-General Edward Sabine, R.A., Treas. and V.P.R.S. Received March 7, 1861.

I propose in this communication to bring together and discuss four determinations at different epochs, in which I have myself been either directly or indirectly concerned, which have had expressly in view the object which forms the title of the paper.

Epoch of 1821.—The experiments on this occasion were made in a part of the Regent's Park, then occupied as the nursery garden of Mr. Jenkins: an unexceptionable locality in all respects, and far

distant at that time from buildings or iron implements, railing, or pipes. The experiments, ten in number, were made on six different days, between the 3rd and 10th of August 1821 ; and all between 8 A.M. and 4 P.M. The circle employed was $11\frac{1}{2}$ inches in diameter, made by Nairne, a celebrated artist in his day for instruments of this description: the needle was made by Dollond on Professor Tobias Meyer's principle, described in the Göttingen Transactions for 1814. The size of the small spheres, or their distance from the needle, was varied in the different experiments, so as to bring different parts of the axle to rest on the agate planes. The mean of the ten experiments was 70° 02′·9 N., corresponding to the epoch 1821.65 : the extremes being 70° 00′·1 and 70° 05′·9. The whole of the experiments were made by myself, and are detailed in a paper in the Phil. Trans. for 1822, Art. 1.

Epoch of 1838.—The experiments on this occasion were made on different days in 1837 and 1838, in the course of the magnetic survey of Great Britain, by Messrs. Robert Were Fox and John Phillips, Captain (since Admiral) Sir James Clark Ross, Captain Edward Johnson of the Royal Navy, and myself. The instruments employed were those of Robinson, Gambey, and Jordan : the particulars are recorded in the 8th volume of the Reports of the British Association for the Advancement of Science (1839), Table 10, p. 64. The localities in which the experiments were made were—1. The same spot in the Regent's Park where those of 1821 had been made. 2. Kew Gardens. 3. Westbourne Green, a locality which has been since built over. Separate determinations were made on 13 days between May 30, 1837, and December 10, 1838, the mean epoch being 1838.3, and the mean dip 69° 17′·3 N. The extremes of all the observers and of all the instruments were 69° 13′·3, and 69° 23′·9.

Epoch of 1854.—The experiments on which this determination rests were made by the late Mr. John Welsh, of the Kew Observatory, and myself in August and September 1854, with two inclinometers made by Mr. Henry Barrow (successor to Mr. Robinson), fitted according to the modern English construction with verniers and microscopes, and each having two needles. The localities selected were—1. The station in the Regent's Park already named as that of the experiments in 1821, and of a part of those in 1838 ; and 2. the magnetic house of the Kew Observatory. The experiments had a

double purpose, viz. 1, to ascertain the difference, if any, in the dip in the Regent's Park and in the magnetic house at Kew; and 2, to obtain a determination of the dip in August 1854 which might be strictly comparable with the result obtained in August 1821. The experiments were made on five different days, and comprised eighteen determinations, ranging between 68° 29′·25 and 68° 33′·73; the mean being 68° 31′·13 N., corresponding to the epoch 1854.65. The mean of eight determinations in the Regent's Park was 68° 30′·55, and of ten determinations at Kew, 68° 31′·6; the difference of either from the mean being 0′·52, which is within the limits of probable error. A detailed notice of these experiments was published in 1855 in an Editor's note in p. 364 of the translation, edited by myself, of Arago's Meteorological Essays.

Epoch of 1859.5.—The dip corresponding to July 1, 1859 (now first discussed), is derived from 282 determinations made in the magnetic house at Kew on 121 different days between November 1857 and December 1860 inclusive, chiefly by four observers, viz. Mr. John Welsh, late Director of the Kew Observatory, Mr. Balfour Stewart, its present Director, Mr. Chambers, Assistant in the Observatory, and Dr. Bergsma, Director of the Magnetical and Meteorological Observatory of the Netherlands Government in Java.

There were employed in these determinations, on different occasions, twelve circles and twenty-four needles, all of the same form and pattern; the circles being 6 inches in diameter, fitted with verniers and microscopes, and the needles 3½ inches in length; they were all made by Mr. Henry Barrow. Every determination was complete in the eight different positions of the circle and needle, as described in Appendix 2 of the Article "Terrestrial Magnetism" in the 3rd edition of the 'Admiralty Manual of Scientific Inquiry.' The individual results are shown in the subjoined Tables, whereof Table I. contains 115 determinations comprised between November 1857 and December 1858; Table II. 96 determinations between January and December 1859; and Table III. 71 determinations between January and December 1860. The results in each year are reduced to the 1st of July in the same year, employing the proportional parts of an annual secular change of —2′·6: those which were obtained in the four winter months, November, December, January, and February, have also received a correction of —0′·8, and those obtained in the summer

months, May, June, July, and August, a correction of $+0'{\cdot}8$ in compensation for annual variation, agreeably to an investigation contained in the sequel. The Tables exhibit in every case the date, the particular circle and needle employed, the azimuths in which the observations were made, the name of the observer, the observed dip, the reduction to a common epoch, the correction for annual variation, and finally, the corrected result.

The opportunity afforded at the Kew Observatory, of testing the degree of accordance which may be expected in the results of different instruments constructed on the plan which has been for several years past approved and adopted at Kew, has thus been profited by, and the conclusions appear such as to merit the consideration of those who are desirous to possess reliable instruments. Several of the circles are the property of foreign governments or of individuals, at whose request they were provided subject to a verification at Kew. The observations here recorded were for the most part made for the purpose of such verifications, and were entered as they were made in the books of the Kew Observatory, from which they are now taken. No observation has been omitted. The circles were distinguished by the numbers 20, 23, 27, 28, 30, 31, 33, 34, 35 and 36; and two unnumbered, one known as the Kew Circle, the other an inclinometer employed by Admiral Sir James Clark Ross in his recent magnetic survey of a part of England. No. 20 was made for Professor Hansteen of Christiania, and is now in his possession; 23 is the circle used by the late Mr. Welsh in his magnetic survey of Scotland; 27 was supplied to the Austrian frigate 'Novara' for her voyage of circumnavigation; 28 was made for the Russian Government; 30 was used by myself in the recent magnetic survey of England, and has been since supplied to the Observatory at the "Isle Jesus" near Montreal in Canada; 31 was made for Padre Secchi of the Collegio Romano, and is now at Rome; 32 was made for the Rev. Alfred Weld of Stonyhurst College, and is now in the Observatory of that College; 34 was supplied to the Government of the United States of America, and is now in the possession of Dr. Alexander Dallas Bache, Superintendent of the Coast Survey; Nos. 35 and 36 were made for the Netherlands Government, one for Utrecht, and one for Java; the "Kew Circle" was in regular use for the monthly determinations of the Dip at Kew, from the commencement of those observations until

August 1859, when it was exchanged for No. 33, which has subsequently been, and is now, in regular employment for that purpose.

Besides the four principal observers already noticed, a few determinations were made, as is shown in the Tables, by Mr. Valentine Magrath, Assistant in the Observatory, by Captain Haig of the Royal Artillery, practising at Kew preparatory to his employment on the Boundary Commission between the United States and the British possessions on the West Coast of North America, and by Lieut. Goodall of the Royal Engineers, who attended at Kew to practise the manipulation of magnetical instruments.

Table I.—Observations of the Magnetic Dip, at the Kew Observatory, in 1857 and 1858, with Circles of the English Construction, fitted with Verniers and Microscopes.

Date.	Circle.	Needle.	Azimuth.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1858.
1857.								
Nov. 2	27	1	0° & 180°	Mr. Welsh.	68° 23′·4	−1′·8	−0′·8	68° 20′·8
2	27	1	30 „ 120	„	24·9	1·8	0·8	22·3
2	27	1	60 „ 150	„	24·5	1·8	0·8	21·9
2	27	2	0 „ 180	„	25·2	1·8	0·8	22·6
2	27	2	30 „ 120	„	23·8	1·8	0·8	21·2
2	27	2	60 „ 150	„	24·0	1·8	0·8	21·4
2	Kew	2	0 „ 180	Mr. Chambers.	27·0	1·8	0·8	24·4
2	Kew	2	30 „ 120	„	24·0	1·8	0·8	21·4
2	Kew	2	60 „ 150	„	24·0	1·8	0·8	21·6
3	23	1	0 „ 180	Mr. Welsh.	26·3	1·8	0·8	23·7
3	23	1	30 „ 120	„	26·6	1·8	0·8	24·0
3	23	1	60 „ 150	„	25·6	1·8	0·8	23·0
3	23	2	0 „ 180	„	26·6	1·8	0·8	24·0
3	23	2	30 „ 120	„	25·5	1·8	0·8	22·9
3	23	2	60 „ 150	„	25·0	1·8	0·8	22·4
3	27	3	0 „ 180	Mr. Chambers.	22·0	1·8	0·8	19·4
3	27	3	30 „ 120	„	23·5	1·8	0·8	20·9
3	27	3	60 „ 150	„	23·5	1·8	0·8	20·9
4	28	1	0 „ 180	„	22·3	1·8	0·8	19·7
4	28	1	30 „ 120	„	25·1	1·8	0·8	22·5
4	28	1	60 „ 150	„	27·2	1·8	0·8	24·6
5	28	2	0 „ 180	„	28·1	1·8	0·8	25·5
5	28	2	30 „ 120	„	25·3	1·8	0·8	22·7
5	28	2	60 „ 150	„	28·1	1·8	0·8	25·5
7	28	1 & 4	Mer., and at right angles.	„	25·6	1·8 1·8	0·8	23·0
10	27	1 & 4	Mer., and at right angles.	„	34·2	1·8 1·8	0·8	31·6
27	28	1	0 & 180	„	24·9	1·4	0·8	22·7
27	28	2	„	„	68 29·4	−1·4	−0·8	68 27·2

TABLE I. (*continued.*)

Date.		Circle.	Needle.	Azimuth.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1858.
1858.									
Jan.	1	Kew	1	0° & 180°	Mr. Chambers.	68° 20'·7	−1'·4	−0'·8	68° 18'·5
	1	Kew	2	,,	,,	24·7	1·4	0·8	22·5
	5	Kew	2	,,	,,	24·6	1·4	0·8	22·4
	5	Kew	1	,,	Mr. Welsh.	20·8	1·4	0·8	18·6
	8	Kew	1	,,	Cap. Bedingfield, R.N.	25·1	1·4	0·8	22·9
	14	Kew	2	,,	Capt. Haig, R.A.	24·9	1·3	0·8	22·8
	14	Kew	2	,,	,,	24·6	1·3	0·8	22·5
	16	Kew	1	,,	,,	23·6	1·3	0·8	21·5
	18	23	1	,,	,,	28·4	1·3	0·8	26·3
	18	23	1	,,	,,	22·8	1·3	0·8	20·7
Feb.	3	?		,,	Mr. Chambers.*	27·2	1·2	0·8	25·2
	4	Kew	1	,,	,,	22·9	1·2	0·8	20·9
	4	Kew	2	,,	,,	26·9	1·2	0·8	24·9
	27	Kew	1	,,	,,	20·3	1·0	0·8	18·5
	27	Kew	2	,,	,,	22·5	1·0	−0·8	20·7
Mar.	1	Kew	1	,,	Mr. Welsh.	24·4	1·0	0·0	23·4
	1	Kew	2	,,	,,	24·3	1·0	0·0	23·3
	4	Kew	2	,,	,,	25·4	0·9	0·0	24·5
	5	Kew	1	,,	,,	22·5	0·9	0·0	21·6
	22	Kew	1	,,	,,	23·7	0·7	0·0	23·0
	22	Kew	2	,,	,,	25·9	0·7	0·0	25·2
	27	Kew	1	,,	,,	23·7	0·7	0·0	23·0
	27	Kew	2	,,	,,	26·1	0·7	0·0	25·4
	27	Kew	1	,,	Mr. Chambers.	21·6	0·7	0·0	20·9
	30	20	1	,,	Mr. Welsh.	25·4	0·7	0·0	24·7
	30	30	1	30 & 120	,,	28·3	0·7	0·0	27·6
	30	30	1	60 ,, 150	,,	23·0	0·7	0·0	22·3
	30	30	1	0 ,, 180	,,	23·2	0·7	0·0	22·5
	30	20	1	30 ,, 120	,,	21·3	0·7	0·0	20·6
	30	30	1	60 ,, 150	,,	25·2	0·7	0·0	24·5
	30	30	2	0 ,, 180	Mr. Chambers.	25·1	0·7	0·0	24·4
	30	30	2	30 ,, 120	,,	26·9	0·7	0·0	26·2
	30	30	2	60 ,, 150	,,	27·0	0·7	0·0	26·3
Apr.	22	Kew	2	0 ,, 180	Mr. Magrath.	23·7	0·5	0·0	23·2
	22	Kew	1	,,	,,	24·0	0·5	0·0	23·5
	27	Kew	1	,,	Mr. Welsh.	20·9	0·5	0·0	20·4
	27	Kew	2	,,	,,	21·7	0·5	0·0	21·2
	28	Kew	1	,,	Mr. Chambers.	22·8	0·5	0·0	22·3
	28	Kew	2	,,	,,	21·7	0·5	0·0	21·2
May	20	Kew	1	,,	,,	22·3	0·3	+0·8	22·8
	20	Kew	2	,,	,,	23·9	0·3	0·8	24·4
	26	Kew	1	,,	Mr. Welsh.	20·5	0·3	0·8	21·0
	26	Kew	2	,,	,,	24·1	0·3	0·8	24·6
	27	Kew	1	,,	Mr. Chambers.	21·6	0·3	0·8	22·1
	27	Kew	2	,,	,,	21·3	0·3	0·8	21·8
	27	Sir J. Ross	1	,,	Mr. Welsh.	25·4	0·3	0·8	25·9
	28	Sir J. Ross	1	,,	,,	26·8	0·3	0·8	27·3
	28	Sir J. Ross	1	30 & 120	,,	20·3	0·3	0·8	20·8
	28	Sir J. Ross	1	60 ,, 150	,,	24·0	0·3	0·8	24·5
June	9	30	1	0 ,, 180	,,	68 21·1	−0·2	+0·8	68 21·7

* Marked "Doubtful."

TABLE I. (*continued.*)

Date.		Circle.	Needle.	Azimuth.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1858.
1858.									
June	9	30	2	0° & 180°	Mr. Welsh.	68° 21′·6	−0′·2	+0′·8	68° 22′·2
	22	Kew	1	,,	Mr. Chambers.	19·9	0·1	0·8	20·6
	22	Kew	2	,,	,,	20·7	0·1	0·8	21·4
	22	Kew	2	,,	,,	19·8	0·1	0·8	20·5
	22	Kew	1	,,	,,	18·2	0·1	0·8	18·9
	22	23	1	,,	Mr. Welsh.	27·4	0·1	0·8	28·1
	22	23	2	,,	,,	23·0	0·1	0·8	23·7
	22	23	2	,,	,,	23·1	0·1	0·8	23·8
	22	23	2	30 & 120	,,	25·2	0·1	0·8	25·9
	22	23	2	60 ,, 150	,,	24·7	−0·1	0·8	25·4
	28	23	2	0 ,, 180	,,	25·1	0·0	0·8	25·9
	28	Kew	1	,,	Mr. Chambers.	23·4	0·0	0·8	24·2
	29	23	2	,,	Mr. Welsh.	25·1	0·0	0·8	25·9
	29	23	1	,,	,,	23·9	0·0	0·8	24·7
	29	23	1	,,	Mr. Chambers.	21·2	0·0	0·8	22·0
July	3	32	1	,,	,,	24·3	0·0	0·8	25·1
	6	32	1	,,	,,	25·5	0·0	0·8	26·3
	6	32	2	,,	,,	22·9	0·0	0·8	23·7
	8	32	2	,,	,,	22·7	+0·1	0·8	23·6
	8	32	1	,,	,,	23·1	0·1	0·8	24·0
Aug.	3	Kew	2	,,	,,	20·1	0·3	0·8	21·2
	10	Kew	1	,,	,,	20·7	0·3	0·8	21·8
	10	Kew	2	,,	,,	21·3	0·3	0·8	22·4
	20	Kew	1	,,	,,	21·7	0·4	0·8	22·9
	20	Kew	2	,,	,,	23·5	0·4	+0·8	24·7
Sept.	16	Kew	1	,,	,,	22·1	0·6	0·0	22·7
	17	Kew	2	,,	,,	20·8	0·6	0·0	21·4
Oct.	13	Kew	1	,,	,,	23·4	0·7	0·0	24·1
	14	Kew	2	,,	,,	24·2	0·7	0·0	24·9
Nov.	4	Kew	1	,,	,,	20·0	0·9	−0·8	20·1
	4	Kew	2	,,	,,	23·4	0·9	0·8	23·5
	5	23	1	,,	,,	25·6	0·9	0·8	28·7
	6	23	2	,,	,,	26·1	0·9	0·8	26·2
	29	23	2	,,	,,	23·0	1·0	0·8	23·2
	30	23	1	,,	,,	24·3	1·0	0·8	24·5
Dec.	10	Kew	2	,,	,,	20·8	1·1	0·8	21·1
	29	31	1	,,	,,	22·3	1·3	0·8	22·8
	29	31	2	,,	,,	68 20·4	+1·3	−0·8	68 20·9

July 1, 1858, mean of 115 observations .... 68 23·2

Table II.—Observations of the Magnetic Dip at the Kew Observatory in 1859.

Date.	Circle.	Needle.	Azimuth.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1859.
1859.					° ′	′	′	° ′
Jan. 4	31	1	0° & 180°	Mr. Chambers.	68 22·5	−1·3	−0·8	68 20·4
4	31	2	,,	,,	21·0	1·3	0·8	18·9
11	Kew	1	,,	Mr. V. Magrath.	23·6	1·3	0·8	21·5
11	30	1	,,	Mr. Chambers.	23·7	1·3	0·8	21·6
11	30	1	,,	,,	24·6	1·3	0·8	22·5
11	Kew	1	,,	,,	21·6	1·3	0·8	19·5
12	30	2	,,	,,	23·7	1·3	0·8	21·6
12	Kew	1	,,	,,	22·2	1·3	0·8	20·1
12	30	2	,,	,,	24·4	1·3	0·8	22·3
24	30	1	,,	,,	22·7	1·2	0·8	20·7
24	Kew	2	,,	,,	19·8	1·2	0·8	17·8
24	30	1	,,	,,	21·6	1·2	0·8	19·6
25	30	2	,,	,,	24·1	1·2	0·8	22·1
25	Kew	2	,,	,,	20·4	1·2	0·8	18·4
25	30	1	,,	,,	22·2	1·2	0·8	20·2
25	Kew	1	,,	,,	18·9	1·2	−0·8	16·9
Mar. 7	Kew	1	,,	,,	19·6	0·8	0·0	18·8
7	Kew	2	,,	,,	20·1	0·8	0·0	19·3
8	30	1	,,	,,	21·7	0·8	0·0	20·9
8	30	2	,,	,,	22·2	0·8	0·0	21·4
8	20	1	,,	,,	20·8	0·8	0·0	20·0
8	20	2	,,	,,	22·8	0·8	0·0	22·0
10	20	2	,,	,,	24·6	0·8	0·0	23·8
10	33	2	,,	,,	20·5	0·8	0·0	19·7
11	20	1	,,	,,	24·1	0·8	0·0	23·3
11	33	1	,,	,,	25·0	0·8	0·0	24·2
11	33	1	,,	,,	22·2	0·8	0·0	21·4
11	33	2	,,	,,	22·3	0·8	0·0	21·5
11	34	1	,,	,,	21·0	0·8	0·0	20·2
11	34	2	,,	,,	21·6	0·8	0·0	20·8
12	34	1	,,	,,	21·4	0·8	0·0	20·6
12	34	2	,,	,,	20·4	0·8	0·0	19·6
12	23	1	,,	,,	25·0	0·8	0·0	24·2
12	23	2	,,	,,	21·8	0·8	0·0	21·0
14	23	1	,,	,,	24·6	0·8	0·0	23·8
14	23	2	,,	,,	22·7	0·8	0·0	21·9
15	33	1	,,	,,	22·9	0·8	0·0	22·1
15	33	2	,,	,,	21·7	0·8	0·0	20·9
15	Kew	1	,,	,,	21·5	0·8	0·0	20·7
15	Kew	2	,,	,,	26·3	0·8	0·0	25·5
16	33	1	,,	,,	22·9	0·8	0·0	22·1
16	33	2	,,	,,	21·8	0·8	0·0	21·0
16	34	1	,,	,,	21·5	0·8	0·0	20·7
16	34	2	,,	,,	19·5	0·8	0·0	18·7
17	34	1	,,	,,	24·8	0·7	0·0	24·1
17	34	2	,,	,,	18·4	0·7	0·0	17·7
17	30	1	,,	,,	21·7	0·7	0·0	21·0
17	30	2	,,	,,	68 24·4	−0·7	0·0	68 23·7

TABLE II. (*continued.*)

Date.	Circle.	Needle.	Azimuth.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1859.
1859.					° ′	′	′	° ′
Mar. 18	20	1	0° & 180°	Mr. Chambers.	68 23·3	−0·7	0·0	68 22·6
18	20	2	„	„	25·0	0·7	0·0	24·3
18	Kew	2	„	„	21·5	0·7	0·0	20·8
19	Kew	1	„	„	21·5	0·7	0·0	20·8
19	20	1	„	„	23·7	0·7	0·0	23·0
19	20	2	„	„	26·7	0·7	0·0	26·0
April 20	23	2	„	Dr. Bergsma.	22·5	0·5	0·0	22·0
?	?	?	„	„	21·2	0·5	0·0	20·7
?	?	?	„	„	19·2	0·5	0·0	18·7
30	33	2	„	„	21·7	0·5	0·0	21·2
May 16	33	1	„	„	18·8	0·3	+0·8	19·3
16	33	2	„	„	18·1	0·3	0·8	18·6
16	Kew	1	„	Mr. Chambers.	19·0	0·3	0·8	19·5
16	Kew	2	„	„	19·4	0·3	0·8	19·9
17	Kew	1	„	Dr. Bergsma.	18·2	0·3	0·8	18·7
17	Kew	2	„	„	20·3	0·3	0·8	20·8
17, 18	33	1	„	Mr. Chambers.	20·7	0·3	0·8	21·2
17, 18	33	2	„	„	20·7	−0·3	0·8	21·2
July 7	23	1	„	„	21·3	0·0	0·8	22·1
7	23	2	„	„	21·9	0·0	0·8	22·7
Aug. 10	Kew	1	„	„	17·4	+0·3	0·8	18·5
10	33	1	„	„	18·3	0·3	0·8	19·4
15	33	2	„	„	21·8	0·3	0·8	22·9
15	Kew	2	„	„	17·2	0·3	0·8	18·3
22	34	1	„	„	25·3	0·3	0·8	26·4
22	Kew	2	„	„	20·6	0.3	0·8	21·7
22	Kew	1	„	„	22·6	0·3	0·8	23·7
22	34	2	„	„	21·9	0·3	+0·8	23·0
Sept. 12	34	2	„	„	21·9	0·6	0·0	22·5
12	34	1	„	„	20·4	0·6	0·0	21·0
21	33	2	„	„	23·6	0·6	0·0	24·2
21	33	1	„	„	22·2	0·6	0·0	22·8
Oct. 29	33	1	„	„	25·6	0·9	0·0	26·5
29	34	2	„	„	24·1	0·9	0·0	25·0
29	34	1	„	„	22·8	0·9	0·0	23·7
29	33	2	„	„	23·7	0·9	0·0	24·6
Nov. 17	33	1	„	„	23·9	1·0	−0·8	24·1
17	34	1	„	„	22·1	1·0	0·8	22·3
17	34	2	„	„	22·4	1·0	0·8	22·6
17	33	2	„	„	24·3	1·0	0·8	24·5
17	34	1	„	Mr. Magrath.	21·0	1·0	0·8	21·2
17	33	1	„	„	24·3	1·0	0·8	24·5
17	33	2	„	„	24·3	1·0	0·8	24·5
17	34	2	„	„	21·2	1·0	0·8	21·4
18	34	1	„	„	20·5	1·0	0·8	20·7
18	34	1	„	Mr. Chambers.	20·1	1·0	0·8	20·3
Dec. 21	34	1	„	„	20·2	1·2	0·8	20·6
21	34	2	„	„	68 21·4	+1·2	−0·8	68 21·8

July 1, 1859, Mean of 96 observations .... 68 21·5

Table III.—Observations of the Magnetic Dip at the Kew Observatory in 1860.

Date.	Circle.	Needle.	Azimuth.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1860.
1860.			° °		° ′	′	′	° ′
Jan. 20	33	1	0 & 180	Mr. Chambers.	68 21·9	−1·2	−0·8	68 19·9
20	33	2	,,	,,	22·9	1·2	0·8	20·9
Feb. 17	33	1	,,	,,	20·7	1·0	0·8	18·9
17	33	2	,,	,,	21·5	1·0	−0·8	19·7
Mar. 16	33	1	,,	,,	20·8	0·8	0·0	20·0
17	33	2	,,	,,	21·2	0·8	0·0	20·4
April 2	33	1	,,	Lt. Goodall, R.E.	21·9	0·7	0·0	21·2
2	33	2	,,	,,	24·7	0·7	0·0	24·0
18	33	2	,,	Mr. Chambers.	22·8	0·6	0·0	22·2
18	33	1	,,	,,	21·6	0·6	0·0	21·0
26	36	1	,,	,,	17·3	0·5	0·0	16·8
27	36	1	,,	Mr. Stewart.	18·5	0·5	0·0	18·0
27	36	2	,,	,,	18·6	0·5	0·0	18·1
27	35	2	,,	,,	20·4	0·5	0·0	19·9
27	35	2	,,	Mr. Chambers.	20·6	0·5	0·0	20·1
27	36	2	,,	,,	14·8	0·5	0·0	14·3
28	35	1	,,	,,	23·9	0·5	0·0	23·4
May 2	36	1	,,	Mr. Stewart.	18·1	0·5	+0·8	18·4
15	35	1	,,	,,	20·5	0·3	0·8	21·0
15	33	1	,,	Mr. Chambers.	19·5	0·3	0·8	20·0
15	33	2	,,	,,	20·5	0·3	0·8	21·0
22	35	1	,,	Dr. Bergsma.	19·2	0·3	0·8	19·7
22	35	2	,,	,,	19·0	0·3	0·8	19·5
23	36	1	,,	,,	18·0	0·3	0·8	18·5
24	36	2	,,	,,	19·7	0·3	0·8	20·2
June 18	33	2	,,	Mr. Chambers.	20·1	0·1	0·8	20·8
18	33	1	,,	,,	19·1	0·1	0·8	19·8
19	36	1	,,	Dr. Bergsma.	16·6	0·1	0·8	17·3
20	35	1	,,	,,	17·7	0·1	0·8	18·4
21	35	1	,,	,,	19·3	0·1	0·8	20·0
21	35	1	,,	,,	18·8	0·1	0·8	19·5
22	35	2	,,	,,	19·2	0·1	0·8	19·9
22	35	2	,,	,,	18·8	−0·1	0·8	19·5
25	35	1	,,	,,	23·2	0·0	0·8	24·0
25	35	1	,,	,,	22·3	0·0	0·8	23·1
26	35	2	,,	,,	18·4	0·0	0·8	19·3
26	35	2	,,	,,	17·8	0·0	0·8	18·6
29	36	2	,,	,,	19·9	0·0	0·8	20·7
29	36	2	,,	,,	15·1	0·0	0·8	15·9
30	36	1	,,	,,	19·5	0·0	0·8	20·3
30	36	1	,,	,,	19·7	0·0	0·8	20·5
July 2	35	1	,,	,,	21·7	0·0	0·8	22·5
2	36	1	,,	,,	22·6	0·0	0·8	23·4
3	36	2	,,	,,	17·7	0·0	0·8	18·5
5	36	2	,,	,,	18·2	0·0	0·8	19·0
5	35	2	,,	,,	22·9	0·0	0·8	23·7
6	36	2	,,	,,	15·4	0·0	0·8	16·2
7	36	2	,,	,,	68 15·7	+0·1	+0·8	68 16·6

TABLE III. (*continued.*)

Date.		Circle.	Needle.	Azimuth.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1860.
1860.									
July	9	35	1	0° & 18°	Dr. Bergsma.	68° 20′·4	+0′·1	+0′·8	68° 21′·3
	9	35	2	,,	,,	17·0	0·1	0·8	17·9
	9	36	1	,,	,,	14·5	0·1	0·8	15·4
	9	36	2	,,	,,	13·2	0·1	0·8	14·1
	23	33	1	,,	Mr. Chambers.	18·6	0·2	0·8	19·6
	23	33	2	,,	,,	21·5	0·2	0·8	22·5
Aug.	16	33	1	,,	,,	16·4	0·4	0·8	17·6
	16	33	2	,,	,,	16·9	0·4	+0·8	18·1
Sept.	14	33	2	,,	,,	19·9	0·6	0·0	20·5
	14	33	1	,,	,,	18·9	0·6	0·0	19·5
Oct.	19	33	2	,,	,,	21·2	0·9	0·0	22·1
	19	33	1	,,	,,	20·5	0·9	0·0	21·4
	22	30	1	,,	Mr. Stewart.	18·0	0·9	0·0	18·9
	22	30	2	,,	,,	20·6	0·9	0·0	21·5
	22	30	2	,,	,,	21·2	0·9	0·0	22·1
	22	30	1	,,	,,	14·9	0·9	0·0	15·8
	23	30	1	,,	,,	17·6	0·9	0·0	18·5
	29	30	1	,,	,,	19·2	0·9	0·0	20·1
	30	30	2	,,	,,	23·0	0·9	0·0	23·9
Nov.	24	33	1	,,	Mr. Chambers.	20·3	1·0	−0·8	20·5
	26	33	2	,,	,,	21·3	1·0	0·8	21·5
Dec.	18	33	2	,,	,,	19·1	1·2	0·8	19·5
	19	33	1	,,	,,	68 18·0	+1·2	−0·8	68 18·4
July 1, 1860, Mean of 71 observations									68 19·8

Correction for Annual Variation.—Wherever, in the middle latitudes of the northern hemisphere, observations of the dip have been made with sufficient care, it has been found that, after elimination of the effects of secular change, the north dip is somewhat greater in winter than in summer. In the 3rd volume of the Toronto Observations, pp. cxxii and cxxiii, the following Table is given as the result of fifteen years of careful observation made throughout at the same spot and according to the same method of observation, comprising 1920 independent determinations nearly equally distributed in the different months, and averaging about 128 determinations for each of the twelve months; by combining the months equidistant from July (or the middle of the year), the influence of secular change is eliminated :—

Mean of January and the following December	75° 18′·90 N.
Mean of February and the following November ..	75 18·98 „
Mean of March and the following October	75 18·63 „
Mean of April and the following September......	75 18·71 „
Mean of May and the following August	75 17·70 „
Mean of June and the following July	75 17·25 „

Hence on the 1st of July the mean dip at Toronto would be derived as follows, viz.:—

From the four winter months, November to February ..	75° 18′·97
From the four summer months, May to August	75 17·47

Showing an excess of 1′·5 in the winter months above the summer months.

The annual variation at Kew, as it may be derived from the 282 determinations in Tables I., II. and III., does not differ materially from this conclusion. There are in these Tables 87 results obtained in the four winter months of the different years, and 93 results obtained in the four summer months of those years. If we collect into separate means the results in the winter months of 1857–58, 1858–59, and 1859–60, numbering them (1), (3), and (5),—and into separate means the results in the summer months of 1858, 1859 and 1860, numbering them (2), (4), and (6),—and if we compare (1) and (3) with (2), (3) and (5) with (4), (2) and (4) with (3), and (4) and (6) with (5), (by which comparisons the effects of secular change are eliminated), we find an excess of 1′·7 in the mean dip of the winter months over that obtained from the summer months. The mean of the two corrections, thus separately obtained at Toronto and Kew, is 1′·6; of which the half, or 0′·8, has been applied in the Tables with the — sign to the results in the winter months, and with the + sign to the results in the summer months.

Probable error of a single determination of the Dip.—It may be desirable to state the probable error of a single determination as it may be derived from the observations in the Tables, before and after the application of the correction for annual variation. It will be seen that the probable error is diminished by the application that has been made of a correction on this account,

	When uncorrected for annual variation.	When corrected for annual variation.
From the 115 Results in Table I. ..	$\pm 1'{\cdot}50$..........	$\pm 1'{\cdot}49$
From the 96 Results in Table II.	$\pm 1{\cdot}44$..........	$\pm 1{\cdot}39$
From the 71 Results in Table III.	$\pm 1{\cdot}57$..........	$\pm 1{\cdot}46$
	$\pm 1{\cdot}50$..........	$\pm 1{\cdot}45$

The probable error thus obtained represents all the diversities ascribable to the employment of different instruments (all of the one construction),—to the supposed peculiarities of different observers, —to the occasional presence of magnetic disturbance (for which no correction has been attempted),—and to differences due to different *hours* of observation;—in addition to what may be more strictly viewed as "observational errors." It may thus serve in some measure as a guide to those engaged in similar researches, as to the degree of accuracy which is attainable in such experimental inquiries, when proper care is taken in the procurement of a reliable inclinometer and in its manipulation.

For the purpose of comparing the probable error thus obtained with inclinometers of the later English pattern with that of the instruments of earlier construction, four of the latter were selected, viz. a 9-inch circle by Robinson, a 9-inch circle by Barrow, and two 6-inch circles by Robinson, all in good order. Each circle was furnished with two needles of the same length as the diameter of the circle, and read by a lens in lieu of verniers and microscopes. Table IV. contains the particulars of 20 determinations made with these instruments in 1860 by Messrs. Stewart and Chambers. Their mean result is 68° 20′·04 reduced to the epoch 1860.5, and corrected for annual variation. The mean result of the 71 determinations at the same epoch in Table III. is 68° 19′·8. There is therefore no notable difference in the mean results obtained by the two classes of instruments; but there is a considerable difference in the probable error; as from the 20 determinations in Table IV. we obtain $\pm 3'{\cdot}65$ as the probable error of a single determination with the instruments of the earlier pattern, whilst $\pm 1'{\cdot}5$ has been shown to be the probable error when inclinometers of the more recent pattern were employed.

TABLE IV.—Observations of the Magnetic Dip, at the Kew Observatory, in 1860, with 9- and 6-inch Circles (Robinson's and Barrow's), without Verniers.

Date.	Circle.	Needle.	Observer.	Observed Dip.	Reduction to Epoch.	Correction for Annual Variation.	Corrected Dip, July 1, 1860.
1860.				° ′	′	′	° ′
Mar. 16	Robinson's 9-inch	1	Mr. Stewart.	68 29·4	−0·8	0·0	68 28·6
17	"	1	"	24·7	0·8	0·0	23·9
17	"	1	"	18·0	0·8	0·0	17·2
19	"	2	"	14·2	0·8	0·0	13·4
19	"	2	"	13·5	0·8	0·0	12·7
19	"	2	"	22·7	0·8	0·0	21·9
21	Barrow's 9-inch	1	"	11·0	0·8	0·0	10·2
21	"	1	"	16·5	0·8	0·0	15·7
21	"	2	"	24·0	0·8	0·0	23·2
21	Robinson's 9-inch	1	Mr. Chambers.	16·6	0·8	0·0	15·8
21	"	2	"	21·8	0·8	0·0	21·0
22	Barrow's 9-inch	2	Mr. Stewart.	26·5	0·8	0·0	25·7
22	"	2	Mr. Chambers.	23·2	0·8	0·0	22·4
22	"	1	"	18·0	0·8	0·0	17·2
May 2	Robinson's 6-inch, No. 1	1	"	26·3	0·5	+0·8	26·6
4	"	2	"	30·7	0·5	+0·8	31·0
4	Robinson's 6-inch, No. 2	1	"	18·1	0·5	+0·8	18·4
4	"	2	"	17·5	−0·5	+0·8	17·8
Nov. 26	"	1	Mr. Stewart.	19·0	+1·1	−0·8	19·3
26	"	2	"	68 18·5	+1·1	−0·8	68 18·8
July 1, 1860, mean of 20 observations							68 20·04

The observations were all made in the plane of the magnetic meridian.

The values obtained for the Dip at the epochs of 1821 and 1854, having been derived from observations made at the close of August and beginning of September in those years, require a small correction for annual variation, to bring them into strict comparison with the values at the two other epochs of 1838 and 1859, which have been derived from observations distributed generally throughout the years. A correction of $+0'{\cdot}5$ has been applied to each on this account.

Corrected Dip at the several Epochs.—We have then the observed Dips, finally corrected, at the several epochs as follows :—

1821.65	70° 03′·4 N.	(1)
1838.3	69 17·3	(2)
1854.65	68 31·6	(3)
1859.5	68 21·5	(4)

Between No. 1 and No. 4 we have an interval of 37.85 years, and a mean annual secular change of $-2'\cdot69$; mean epoch, 1840.6.

Between No. 1 and No. 2, comprising an interval of 16.65 years, we have a mean secular change of $-2'\cdot77$; mean epoch, 1830.0.

Between No. 2 and No. 4, comprising an interval of 21.2 years, we have a mean secular change of $-2'\cdot63$; mean epoch, 1848.9.

Hence we may infer that the yearly diminution of the Dip from secular change, though very nearly uniform throughout the whole interval of 37.85 years, was somewhat greater in the earlier part of the interval than in the later; and that the rate of diminution may admit of being more exactly represented by the introduction of a second term.

If then we take the year 1840.0 as a convenient middle epoch$=t_0$, and call its dip θ_0; and if we further call the observed dip at the several observational epochs t_1, t_2, t_3 and t_4, respectively θ_1, θ_2, θ_3, θ_4, we shall have four equations of the form

$$\theta_1=\theta_0+x(t_1-t_0)+y(t_1-t_0)^2;$$

and giving double weight to the equation furnished by the epoch 1859.5, inasmuch as it is derived from so much greater a body of observations than the results at the other three epochs, we obtain by least squares,

$$\theta_0=69^\circ\ 11'\cdot95; \qquad x=-2'\cdot713; \qquad y=+0'\cdot00057.$$

Hence we have the general formula for computing the dip between the years 1820 and 1860,

$$\theta=69^\circ\ 11'\cdot95-2'\cdot713\ (t-t_0)+0'\cdot00057\ (t-t_0)^2,$$

t_0 being 1840.0, and t being any other time for which the dip θ is required.

Using this formula, we have the differences between the computed and the observed dips at the several epochs of observation as follows:—

	Computed.	Observed.	Computed—Observed.
1821.65	70° 03'·6	70° 03'·4	+0'·2
1838.3	69 16·8	69 17·3	−0·5
1854.65	68 33·4	68 31·6	+1·8
1859.5	68 21·2	68 21·5	−0·3

And the dips corresponding to every tenth year within the period specified are as follows:—

1820.0	....	70° 07′·3
1830.0	....	69 39·6
1840.0	....	69 11·9
1850.0	....	68 45·9
1860.0	....	68 19·9

The progressive diminution of the Dip in London during the last forty years has thus been traced and followed by the observations recorded and discussed in this paper; and the further progress of the research will now devolve on the systematic observations which are made for that purpose monthly at the Observatory at Kew.

The rate of diminution in the last forty years does not appear to differ materially from the mean rate in the preceding hundred years. The experiments of Mr. George Graham between March and May, 1723, recorded in the Philosophical Transactions for 1725, No. 389, give a mean dip in London at that epoch of "nearly" 74° 40′. Comparing this with 69° 11′·95 in 1840.0, we have a difference of 5° 28′·1 in 116·7 years, equivalent to a uniform diminution of 2′·81 annually; or if the formula

$$\theta = 69^\circ\ 11'{\cdot}95 - 2'{\cdot}713\ (t-t_0) + 0'{\cdot}00056\ (t-t_0)^2$$

be employed, it gives the dip in March 1723.3 equal to 74° 36′·1, being a difference of less than 4′ from the result of Mr. Graham's experiments; which difference is doubtless less than the probable error of that gentleman's determination with the instruments then in use.

An expectation appears to have prevailed in some quarters that the decrease of the Dip in London should have ceased, and its subsequent increase have commenced, contemporaneously with the alteration which took place in the secular change of the *Declination* in the early part of this century, when the increase of west declination, which had been continuous in the British Islands for about two centuries, ceased, and was succeeded by a decrease of the same. But this supposition is by no means in accordance with that general view and interpretation of the phenomena of terrestrial magnetism for which we are indebted to Dr. Halley, and which, since its promulgation in 1683, has received so much confirmation in various and distant parts of the globe. In accordance with that hypothesis, the

diminution of the Dip in London might be expected to continue until the epoch should arrive when, by the easterly movement of translation of the minor magnetic system in the northern hemisphere, the disparity of the magnetic force prevailing in the European and American portions of the hemisphere should have attained its maximum:—which is certainly not yet the case.

Is there then, in the secular change of the Dip, no feature in which, in conformity with the Halleian hypothesis, an alteration might be expected to synchronize with the reversal in the direction of the secular change of the declination? Assuredly there is; and the facts which recent investigations have brought to our knowledge manifest that such an alteration has taken place. I proceed to describe it.

If we have recourse to those extensive generalizations which, under the name of "Isoclinal Lines corresponding to particular Epochs," present a connected view of the changes which have taken place from time to time in the magnetic lines of the Dip over large portions of the earth's surface, and enable us to anticipate with some degree of confidence the changes which may be expected to take place in years to come, we notice generally that the lines undergo two species of modification, or peculiarities of change, which it is necessary to keep separately and distinctly in view. In the British Islands, for example, the Isoclinal Lines for little less than two centuries past have been steadily advancing towards the north by a gradual movement of translation. This is one feature of the secular change; but there is a second feature, which, if not at first sight equally striking, is yet equally regular and systematic in its operation; viz. the *direction* of the isoclinal lines as they pass across our country from the south-west towards the north-east undergoes a small but sensible change from year to year, by which, in the lapse of several years, the angle at which they cut the geographical meridians is materially altered. By the joint operation of these two processes, the *general configuration of the lines* over large portions of the earth's surface, as well as *their values in particular localities*, are both subject to systematic alteration; a remark which is not limited to the isoclinal lines alone, but is the case also in the isogonic and isodynamic lines. Those who are conversant with Dr. Halley's writings, will be aware that,—in correspondence with his views,—between the epochs when the Dip in London should

attain, respectively, the maximum and the minimum amount which constitute its limits under the system of secular change, an *intermediate* epoch might be anticipated, when the isoclinal lines passing across the British Islands should attain their least angle of inclination to the geographical meridian; towards which they should have progressively advanced, and from which they would as progressively recede. Now, if we compare the line of 70° of dip in the Isoclinal Map of 1780 of the Magnetismus der Erde with that of 1840 in Mr. Keith Johnstone's Physical Atlas, plate 23, we may fix on a point in about 42° North Latitude and 30° West Longitude, in which the Dip has remained nearly stationary, and through which the line of 70° of Dip passed, at both epochs; and we may perceive that, in its easterly course from that point or pivot, this line passed in 1780 *through the middle of France considerably to the South of Paris* (where the Dip was then between 71° and 72°); whereas in 1840 it passed *across England considerably to the north of London* (where the Dip had diminished to little more than 69°). Therefore in the sixty years which had elapsed between the two epochs, 1780 and 1840, the direction of the lines as they impinged upon Western Europe had become much less inclined to the geographical meridian (*i. e.* forming a greater angle with the parallels of latitude) in 1840 than in 1780: and if we consult still earlier maps, we find that a change in the same direction had been progressive from a still earlier period. The particular year in which this feature attained its limit, and an opposite change commenced, cannot now perhaps be precisely determined; it was probably somewhat earlier than 1840. But from the comparison of the magnetic surveys of the British Islands in 1836–37 and 1857–58, it is certain that the change in the direction of the isoclinal lines in this part of the globe has entered upon the contrary phase to that which had previously existed. The observations of the late Mr. Welsh in Scotland in 1857–58 (Brit. Assoc. Reports, 1859), when compared with those of the Scotch Survey made in 1836–37, published in the British Association Reports for 1836, show, according to Mr. Balfour Stewart's calculation, that an increase of several degrees in the angle at which the lines cut the meridians in passing across Scotland has taken place between the epochs of the earlier and the later surveys. The same general conclusion follows from a comparison of the magnetic surveys of England at nearly the same epochs; every-

where near the west coast of England the mean annual secular change in the twenty years has been greater, and near the east coast less than its mean value at Kew; showing that the general direction of the isoclinal lines more nearly approaches a parallelism to the equator now than it did twenty years ago. The ascertainment of the exact value of the secular change at a particular locality by a well-conducted system of periodical observations is the duty of a magnetic *observatory*; the direction of the magnetic lines passing across a country is supplied by magnetic *surveys*; which, for that purpose, ought to be repeated from time to time, as they have now been in this country, at intervals of perhaps twenty or twenty-five years.

It has been imagined that the secular changes of the magnetic elements may be due to some alteration taking place either in the distribution or in the condition of the materials in the interior of the globe. But the regularity and uniformity with which the secular magnetic changes continue through long intervals of time, together with their sudden periodic reversals,—and their corresponding features in the northern and southern hemispheres, which add greatly to the apparent consistency and systematic character of the whole as parts of a uniform general system,—wear more the aspect of effects of some yet unascertained *cosmical* cause. One of the British Colonial Observatories, St. Helena, having the advantage of both a large secular change and a small amount of magnetic disturbance, has afforded a very striking example of the great regularity with which the secular change takes place, maintaining a steady uniformity, traceable not only from year to year, but from month to month, and even from week to week; so that it is not too much to say that, from observations made during a single fortnight, an annual secular change which has existed almost without variation for more than a century, may be ascertained and measured with very considerable precision. (Magnetic Observations at St. Helena, vol. ii. p. ix.)

March 21, 1861.

Major-General SABINE, R.A., Treasurer and Vice-President, in the Chair.

Professor Herman Helmholtz, elected a Foreign Member, was admitted into the Society.